# Weak Bisimulation up to Elaboration

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CONCUR'06
August 30, 2006
Bonn, Germany

## Outline

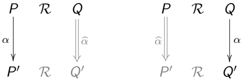
- Weak bisimilarity, Expansion, Elaboration;
- Weak bisimulation up to Elaboration;
- Up to transitivity.

## Weak Bisimilarity

- LTS: processes ($P, Q$..), labelled transitions ($P \xrightarrow{\alpha} P'$).
- Weak transitions:

$$\stackrel{\widehat{\tau}}{\Rightarrow} \triangleq \xrightarrow{\tau}^{\star} \qquad \stackrel{\widehat{a}}{\Rightarrow} \triangleq \xrightarrow{\tau}^{\star}\xrightarrow{a}\xrightarrow{\tau}^{\star} .$$

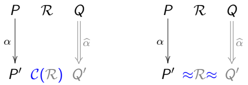
- Weak bisimulation:




- Weak bisimilarity: $\approx \triangleq \bigcup\{\mathcal{R},\ \mathcal{R}$ weak bisimulation $\}$.

## Up-to Techniques

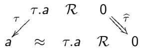
- "Reduce the size of the bisimulation candidate."
- Up to context, up to $\approx$




- The latter technique is not correct!

$\mathcal{R} \triangleq \{\langle \tau.a, 0\rangle\}$

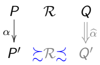
$a \approx \tau.a$



## Expansion [Arun-Kumar, Hennessy - Milner, Sangiorgi - 92]

- Behavioural preorder, $\gtrsim \subseteq \approx$ $\qquad \xrightarrow{\hat\tau} \triangleq \xrightarrow{\tau}^{=} \qquad \xrightarrow{\hat a} \triangleq \xrightarrow{a}$

- $P \gtrsim Q$ : $Q$ must move "faster" than $P$, at each step.
- Weak bisimulation up to expansion:



($\mathcal{R}$ symmetric)

- This technique has been used in complex proofs ($\pi$, Join...).

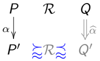
## Elaboration [Arun-Kumar, Natarajan - 95]

- Behavioural preorder, $\gtrapprox \subseteq \approx$ $\qquad \xRightarrow{\tau} \triangleq \xrightarrow{\tau}^{+} \qquad \xRightarrow{a} \triangleq \xRightarrow{\hat a}$

- $P \gtrapprox Q$ : $P$ must move "slower" than $Q$, at each step.
- $\gtrapprox$ and $\gtrsim$ are not comparable: $a \;\overset{\gtrsim}{\not\gtrapprox}\; a|\tau \;\overset{\not\gtrsim}{\gtrapprox}\; \tau.a$
- Elaboration in CCS:
  - Close to $\approx$: if $P \approx Q$, then $!\tau|P \gtrapprox Q$.
  - Contains *progressing bisimulation*: the largest bisimulation which is a congruence [Montanari, Sassone - 91].
  - Almost a precongruence (preemptive power of $\tau$).

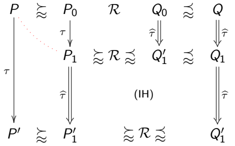
## Weak Bisimulation up to Elaboration?

If $\mathcal{R}$ is symmetric and



then $\gtrapprox \mathcal{R} \lessapprox$ is a bisimulation, and hence $\mathcal{R} \subseteq \approx$

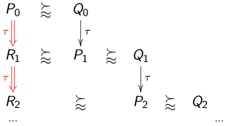
*Proof.*



- $P > P_1$ for some well-founded partial order $>$.
- If $\xrightarrow{\tau}$ terminates, then so does $\gtrapprox \xrightarrow{\tau}$.

## Termination Hypothesis

- If $\xrightarrow{\tau}$ terminates, then so does $\gtrapprox \xrightarrow{\tau}$.



- The same reasoning does not hold with $\approx$.
- **Theorem.** *If $\xrightarrow{\tau}$ terminates, then weak bisimulation up to elaboration is correct.*

## Up to Transitivity (for strong bisimilarity)

If $\mathcal{R}$ is symmetric and $\begin{array}{ccc} P & \mathcal{R} & Q \\ {\scriptstyle\alpha}\downarrow & & \downarrow{\scriptstyle\alpha} \\ P' & \mathcal{R}^\star & Q' \end{array}$ then $\mathcal{R}^\star$ is a strong bisimulation.

- Subsumes the "strong up to strong" technique:

  If $\begin{array}{ccc} P & \mathcal{S} & Q \\ {\scriptstyle\alpha}\downarrow & & \downarrow{\scriptstyle\alpha} \\ P' & \sim\mathcal{S}\sim & Q' \end{array}$ then $\mathcal{R} \triangleq \mathcal{S} \cup \sim$ satisfies the diagram above, and $\mathcal{S} \subseteq \mathcal{R} \subseteq \sim$ .

- Not correct with weak bisimilarity games!
- Only correct on one side for expansion:

  If $\begin{array}{ccc} P & \mathcal{R} & Q \\ {\scriptstyle\alpha}\downarrow & & \downarrow{\scriptstyle\widehat{\alpha}} \\ P' & \mathcal{R}^\star & Q' \end{array}$ and $\begin{array}{ccc} P & \mathcal{R} & Q \\ {\scriptstyle\widehat{\alpha}}\Downarrow & & \downarrow{\scriptstyle\alpha} \\ P' & \mathcal{R} & Q' \end{array}$ then $\mathcal{R} \subseteq \succsim$ .

## Up to Transitivity for Elaboration

- **Theorem.** *If $\xrightarrow{\tau}$ terminates, then up to transitivity is correct for elaboration, i.e.:*

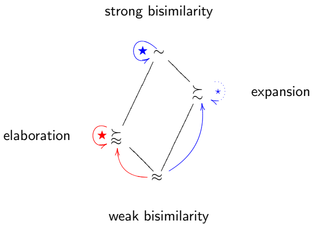
  If $\begin{array}{ccc} P & \mathcal{R} & Q \\ {\scriptstyle\alpha}\downarrow & & \Downarrow{\scriptstyle\widehat{\alpha}} \\ P' & \mathcal{R}^\star & Q' \end{array}$ and $\begin{array}{ccc} P & \mathcal{R} & Q \\ {\scriptstyle\alpha}\Downarrow & & \downarrow{\scriptstyle\alpha} \\ P' & \mathcal{R}^\star & Q' \end{array}$ then $\mathcal{R} \subseteq \gtrapprox$ .

  *Proof sketch.* Prove that $\mathcal{R}^\star \stackrel{\tau}{\Rightarrow}$ terminates, then reason by well-founded induction.

- **Corollary.** (a variant of Newman's Lemma for $\approx$):

  *If $\xrightarrow{\tau}$ terminates and* $\begin{array}{ccc} P & \xrightarrow{\tau} & Q \\ {\scriptstyle\alpha}\downarrow & & \Downarrow{\scriptstyle\widehat{\alpha}} \\ P' & \xRightarrow{\widehat{\tau}} & Q' \end{array}$ *then* $\xrightarrow{\tau} \subseteq \approx$ .

## A Picture




(when $\xrightarrow{\tau}$ terminates)

## Concluding Remarks

- [ICALP'05]: "if $\mathcal{B}^\star$ is a bisimulation and $\mathcal{B}^{+}\xrightarrow{\tau}^{+}$ terminates, then $\mathcal{B}^\star$ is a correct up to technique for $\approx$".
  - The termination hypothesis mentions the relation $\mathcal{B}$.
  - Lacks compositionality: we cannot define the "largest $\mathcal{B}$".
- The up to elaboration technique appears as a much more tractable instantiation of the above results:
  - coinductively defined,
  - up to transitivity, up to context.
- Is it reasonable to ask for the termination of $\xrightarrow{\tau}$?
  - For most algorithms, termination is part of the correctness.
  - When loops are needed (busy waiting), isolate terminating silent actions.
- We still need to demonstrate the use of these techniques on some non-trivial examples.